Differential Subordination and Superordination of Analytic Functions Defined by Cho-Kwon-Srivastava Operator

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Abstract
Differential subordination and superordination results are obtained for analytic functions in the open unit disk which are associated with Cho-Kwon-Srivastava operator. These results are obtained by investigating appropriate classes of admissible functions. Some of the result established in this paper would provide extensions of those given in earlier works.

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Summary of the paper
In the present investigation, the differential subordination result of Miller and Mocanu is extended for analytic functions in the open unit disk, which are associated with Cho-Kwon-Srivastava operator \( I_p(a,c,f(z)) \) and we obtain certain other related results. In the first section of the paper we prove subordination results involving the Cho-Kwon-Srivastava operator \( I_p(a,c,f(z)) \).

Theorem 1. Let \( \phi \in \Phi_I[\Omega,q] \). If \( f(z) \in A(p) \) satisfies
\[
\left\{ \phi \left( I_p^\lambda (a+1,c)f(z), I_p^\lambda (a,c)f(z), I_p^\lambda (a-1,c)f(z); z \right) : z \in U \right\} \subset \Omega,
\]
then
\[
I_p^\lambda (a+1,c)f(z) \prec q(z)
\]

Theorem 2. Let \( \phi \in \Phi_{1,1}[\Omega,q] \). If \( f(z) \in A(p) \) satisfies
\[
\phi \left( \frac{I_p^\lambda (a+1,c)f(z)}{z^{p-1}}, \frac{I_p^\lambda (a,c)f(z)}{z^{p-1}}, \frac{I_p^\lambda (a-1,c)f(z)}{z^{p-1}}; z \right) \prec h(z),
\]
then
\[
I_p^\lambda (a+1,c)f(z) \prec q(z)
\]

The dual problem of the differential subordination, that is, differential superordination of the operator \( I_p^\lambda (a,c,f(z)) \) is investigated in this section. In the this section we prove Superordination of the Cho-Kwon-Srivastava operator \( I_p^\lambda (a,c,f(z)) \).

Theorem 3. Let \( \phi \in \Phi_I[\Omega,q] \). If \( f(z) \in A(p) \). \( I_p^\lambda (a,c,f(z)) \in Q_0 \) and
\[
\phi \left( I_p^\lambda (a+1,c)f(z), I_p^\lambda (a,c)f(z), I_p^\lambda (a-1,c)f(z); z \right)
\]
is univalent in \( U \), then
\[
\Omega \subset \left\{ \phi \left( I_p^\lambda (a+1,c)f(z), I_p^\lambda (a,c)f(z), I_p^\lambda (a-1,c)f(z); z \right) : z \in U \right\}
\]
implies
\[
q(z) \prec I_p^\lambda (a+1,c)f(z).
\]
Theorem 5. Let $\phi \in \Phi^{\prime,1}_{3,3} [\Omega, q]$. If $f(z) \in A(p), \frac{I^\lambda_p(a+1,c)f(z)}{z^{p-1}} \in Q_0$ and
\[
\phi \left( \frac{I^\lambda_p(a+1,c)f(z)}{z^{p-1}}, \frac{I^\lambda_p(a,c)f(z)}{z^{p-1}}, \frac{I^\lambda_p(a-1,c)f(z)}{z^{p-1}}; z \right)
\]
is univalent in $U$, then
\[
\Omega \subset \left\{ \phi \left( \frac{I^\lambda_p(a+1,c)f(z)}{z^{p-1}}, \frac{I^\lambda_p(a,c)f(z)}{z^{p-1}}, \frac{I^\lambda_p(a-1,c)f(z)}{z^{p-1}}; z \right) : z \in U \right\}
\]
implies
\[
q(z) \prec \frac{I^\lambda_p(a+1,c)f(z)}{z^{p-1}}.
\]

Theorem 6. Let $q(z) \in H_0, h(z)$ be univalent in $U$ and $\phi \in \Phi^{\prime,1}_{3,3} [\Omega, q]$. If $f(z) \in A(p), \frac{I^\lambda_p(a+1,c)f(z)}{z^{p-1}} \in Q_0$ and
\[
\phi \left( \frac{I^\lambda_p(a+1,c)f(z)}{z^{p-1}}, \frac{I^\lambda_p(a,c)f(z)}{z^{p-1}}, \frac{I^\lambda_p(a-1,c)f(z)}{z^{p-1}}; z \right)
\]
is univalent in $U$, then
\[
h(z) \prec \phi \left( \frac{I^\lambda_p(a+1,c)f(z)}{z^{p-1}}, \frac{I^\lambda_p(a,c)f(z)}{z^{p-1}}, \frac{I^\lambda_p(a-1,c)f(z)}{z^{p-1}}; z \right)
\]
implies
\[
q(z) \prec \frac{I^\lambda_p(a+1,c)f(z)}{z^{p-1}}.
\]